Quantifiers, Scope, and Pseudo-Scope: When a man loves a woman …

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Abstract

Generalized Quantifier Theory (GQT) is discussed as an analysis of quantifying expression (QE) in natural language. According to that theory, a quantifier denotes a relation between sets, and quantifying DPs are QE which are defined as sets of sets. Such QEs require quantifier raising (QR) and an additional mechanism that provides for the binding of bound variable pronouns. Various restrictions for scope taking in sentences with two and more quantifiers seem to apply; an alternative to scope taking via QR is scope independence, which can be modelled by choice functions or Skolem functions and is informally described in terms of Game Theory. Various data seem to require scope extension in the sense that semantic scope is wider than syntactic scope via c-command. Such extensions are discourse representation theory (DRT) and dynamic logic. Further extensions of GQT deal with pluralities.

Keywords: Generalized quantifiers, generalized quantifier theory, scope, quantifier raising, binding, scope independence, scope inversion, choice functions, game theory, scope extensions

1 Basic notions

1.1 Quantifying expressions and scope

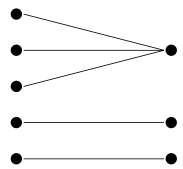
This article tries to bring together two notions, quantification and scope, neither of which is, in and by itself, easy to define: in fact, I do not know of any sufficiently general and precise definition that captures the intuitive content of these terms. I will therefore proceed by way of illustration, by analysing standard examples that exhibit interaction of scope and quantifiers.

Frequently cited in this context (most often in introductory texts by philosophers of logic and language) is sentence (1),

(1) Every man loves a woman

and it is explained that (1) is potentially ambiguous. In order to clarify the ambiguity, one resorts to the native speaker's intuitions about the truth conditions of (1). In one meaning, (1) is true of a situation in which one can find for each man a possibly different woman such that that man loves that woman. Call this the *every-some-reading*. On a second interpretation, call it the *some-every-reading*, (1) is true only if there is a particular woman such that every man loves that woman. A sentence S is ambiguous if it has two different meanings, and two meanings differ if we can find a situation of which S is true in one meaning but false in the other. The typical situation verifying (1) in the every-some-reading, e.g. (2),

(2) men women



would falsify (1) in the some-every-reading. Hence, (1) can be judged true and false in the same situation, depending on the reading of (1). This proves that (1) is ambiguous. The source of the ambiguity is then traced back and explained as a difference in the scope of the quantifiers, which will be the subject of discussion in what follows.

As most readers will know there is a branch of philosophy and mathematics, namely Predicate Logic, that deals with the precise meaning of quantifiers. The quantifiers in (1) are *every* and *a*, the nouns *man* and *woman* are called the respective **restrictions** of the quantifiers. The quantifiers themselves are often denoted by $\forall$ (the universal quantifier "every") and $\exists$ (the existential quantifiers "some" or "a" in natural language). Let us for further reference abbreviate the every-some-reading as $\forall\exists$R and the some-every-reading as $\exists\forall$R. Linguists and philosophers describe the ambiguity of (1) by translating (1) into the two expressions of First Order Predicate Logic shown in (3):

(3) a. $\forall x(\text{man}(x) \rightarrow \exists y(\text{woman}(y) \wedge \text{love}(x, y)))$
 b. $\exists y(\text{woman}(y) \wedge \forall x(\text{man}(x) \rightarrow \text{love}(x, y)))$

These formulae clearly differ in truth conditions and in the order of quantifiers, hence our abbreviations $\forall\exists$R and $\exists\forall$R. In this article, I will only rarely use such formulae; although they are very useful as a tool for precisely stating truth conditions, they are of little avail when it comes to define the nature of quantifiers in natural language.

Taking (1) as a point of departure, let me introduce a bit of terminology. First, we want to say that *every man* and *a woman* are expressions of a certain syntactic type, namely determiner phrases (abbreviated as DPs, formerly aka NPs in syntax). Such quantifying DPs will be abbreviated by **QE**, which is a shorthand for **quantifying expression**. As we will see in section 5, quantification can also be expressed by all sorts of other syntactic categories; I will restrict myself here to QEs of the DP-type.

In general, QEs may have **scope**. Applied to the example at hand, we have the following analyses: in the first interpretation (the $\forall\exists$R) the QE *every man* (also called a *universal QE*) has **wide scope** with respect to *a woman*, or equivalently, *a woman* has **narrow scope** with respect to *every man*, and conversely for the $\exists\forall$R, where *a woman* has **wide scope** with respect to *every man*. Saying that A has scope over B is normally understood as saying that A has *wide* scope with respect to B.

Of course, the above terminology does not really tell us anything about the way the ambiguity may arise. What is the general nature of quantifiers and of scope that might explain the ambiguity? Let us begin with the notion of scope. This can be illustrated by analogy to mathematical examples like 5 + 3 – 6 which can be given two interpretations:

(4) a. ((5 – 3) + 6)
  b. (5 – (3 + 6))

For (4a) the result is 8, the result for (4b) is –4. Of course these different results come about by different syntactic structures, resulting in a difference of scope: addition has wide scope in (a) but narrow scope in (b) (with respect to substraction). So what matters is the order of applying different operations, which in turn is determined by the structure expressed by the bracketing. The notion of scope thus seems to be derived from a syntactic feature of (4), i.e. its bracketing.

How can we transfer this mathematical example to the linguistic example at hand? The sentence under discussion does not appear to be syntactically ambiguous, hence would not qualify for divergent bracketings, nor is there any lexical ambiguity. Nonetheless speakers seem to have intuitions about scopal differences. We conclude from this that there must be some *semantic* notion of scope which guides these intuitions. As in example (4), these intuitions apparently rest on some order of interpretative processes: It seems that interpreting the $\exists\forall$R demands to first choose a particular women, this choice being made before we interpret *every man*. The effect will be that the woman chosen must be one being loved by every man. This order of interpretation seems to correspond to (4b) when first doing addition before substraction. This is opposite to the $\forall\exists$R which tells us to first interpret *every man* by considering every entity x that is a man, and then choose among the women one for each x being loved by x. Hence, scope is a matter of interpreting QEs in a certain order.

I will not go much further towards a general characterization of scope; in particular, I will limit myself to illustrations of scope that involve only two QEs. It is obvious that the notion itself is much broader, as becomes clear with examples like (5) and its paraphrases in (5a) and (5b):

(5) John decided to marry on Tuesday
  a. John decided that Tuesday be the day of his marriage
  b. John's decision to marry was taken on Tuesday

In the (a)-reading, *on Tuesday* is in the scope of *decide*, which is not the case in the (b)-reading. This ambiguity is called **structural**, as it is a consequence of the two possible structures indicated in (6):

(6) a. $\boxed{\text{John}\ \boxed{\text{decided}\ \boxed{\text{to}\ \boxed{\text{marry on Tuesday}}}}}$

  b. $\boxed{\text{John}\ \boxed{\boxed{\text{decided}\ \boxed{\text{to marry}}}\ \text{on Tuesday}}}$

Structural ambiguities are syntactic ambiguities; thus the semantic ambiguity is an immediate consequence of a compositional interpretation of syntactic structures. By contrast, the surface structure of (1) seems to be unambiguous. Nonetheless, we will see later that surface syntax can be enriched by some sort of "transformations" that produce different structures from an underlying presentation, structures that make for unambiguous representations of the different readings. In other words, semantics will not operate directly on surface structure, but rather on an additional representation, known as **Logical Form** (LF), which will be discussed further below in section 3. The main purpose of such "Transparent Logical Forms" is a syntactically unambiguous representation of scope, akin to the use of brackets in (4). However, this does not imply that the very notion of scope has become a purely syntactic one. Although historically, the derivation of Logical Forms was intended as a matter of syntax proper and did not always yield unambiguous representations (cf. May (1977)), semanticists further developed the theory away from syntax, so that the intuitions guiding the constructions of such logical representations overwhelmingly rest on semantic considerations: LFs syntactically mirror an underlying intuition about semantic scope.

Coming back to the nature of quantification, the most accurate definition of QEs might be a purely negative one, namely that QEs are semantically contentful non-referential DPs. That is, they cannot be used to refer to particular entities of the universe. This characterization of course involves the notion of reference, which is a philosophical one we cannot discuss here. The paradigm case of a referring DP is a proper name. Thus, proper names are not QEs, and it so happens that they are scopeless, i.e. they cannot enter into scopal relations with other expressions. To illustrate, *Berta* in (7a) is, in some sense, not in the semantic scope of *everyone*, because Berta cannot receive a scope dependent interpretation. And conversely, *someone* is not in the semantic scope of *Berta* in (7b) because *Berta* cannot induce a scope dependent interpretation on *someone*.

(7) a. Everyone loves Berta
    b. Berta loves someone

Note that our use of scope slightly contrasts with a purely syntactic notion of scope, which equates scope with c-command (at the level of LF, cf. section 3.3 below). Hence, *someone* is in the *syntactic* scope of *Berta* in (7b), but intuitively this notion of scope is irrelevant on the semantic level, hence we would like to say that *someone* is not in the semantic scope of *Berta*.

On the other hand, some theories, e.g. classical **Discourse Representation Theory** (DRT, cf. Kamp and Reyle (1993)), analyse proper names by way of introducing a bound variable, so that (7) would now read as:

(8) a. there is an x, x = Berta, and everyone loves x
    b. there is an x, x = Berta, and x loves someone

This way, names are like QEs which always have wide scope (syntactically and semantically). This move does not affect truth conditions but the way coreference and binding relations are handled in that theory. Hence, what we would call scopeless can formally (i.e. truth conditionally) be equivalent to a wide scope analysis.

1.2 Specificity and logical independence

Interestingly, sentences like (1), although often cited as exhibiting a scopal ambiguity, are far more problematic than one might think. Two objections can be raised against a scopal ambiguity of (1).

First, it has often been observed that indefinite DPs like *a woman* may have two different interpretations: either as genuine QEs (compare also "Generics" in this handbook), or as referential **specific** DPs (cf. Fodor and Sag (1982) and "Kinds of (Non-)Specificity" in this handbook). As quantifiers, they are non-specific, but as specific DPs the speaker/hearer must have a specific person in mind whose identity is irrelevant to the current purpose of communication. The ambiguity would then arise from two different uses of the indefinite *a woman*: either as a real QE in the scope of *every man* or as a scopeless specific indefinite DP. As a referential expression, the specific indefinite's denotation depends, in part, on the intention of the speaker, not on linguistic environments. But otherwise the indefinite would behave like a name.

This notwithstanding, the truth conditions for such a scopeless reading would, as in the case of names in DRT, coincide with the ∃∀R with maximally wide scope for *a woman*. The point I want to bring home here is that this effect can be derived without assuming any syntactic ambiguities.

It seems, therefore, that in the case of (1) we could dispense with transformations and a scopal ambiguity in favor of surface syntax and the specific/non-specific divide. However, we will argue in this section that, although it is important to recognise this distinction, it does not solve the problem: It is still plausible to assume a scopal ambiguity for examples of a slightly different kind, namely those that I will call inverse readings further below.

A quite different second argument that has been raised against a structural ambiguity of (1) relates to the fact that the ∀∃R and the ∃∀R are not logically independent from each other. In fact, every situation truthfully described by the ∃∀R could also model the ∀∃R, if only the woman chosen to verify the ∀∃R is (accidentally) the same for each man. Thus, any verifying situation for one reading is also a possible situation for the other reading. In logical terms this means that one reading *entails* the other. But in such a situation the question arises whether the stronger reading is a genuine reading and not just a description of a special case, the general case being covered by the weaker reading. It is not a priory clear whether the evidence adduced in favor of a stronger reading really suffices to make up for a "reading" in the first place; cf. also the discussion in Ruys and Winter (2011).

I have nothing to offer here that might clarify the issue and what should count as a genuine "reading". Rather, I would like to point out that one of the earliest formally analyzed examples of an ambiguity of sorts was not (1) but:

(9) A woman loves every man
(from Montague (1970a, p. 204) and Montague (1973, p. 268))

Montague claimed that (9) can have the ∀∃R, so that *a woman* is interpreted as being scope-depending on the object *every man*. Let us call this reading, and any reading that does not comply with the (linear) structure of a sentence, an **inverse reading**. The effect of the shift from (1) to (9) is that in (1) the putative inverse scope reading is the one which isn't clearly a separate reading (as just outlined), while in (9), the inverse scope reading must be a real reading, since it doesn't entail the surface scope reading. Obviously, in the case of (9), we cannot get the inverse reading simply by alluding to the difference between *a woman* as either specific or non-specific. Even as non-specific, it is not automatically in the scope of *every man*, so that the inverse reading seems to require some logical representation different from the surface syntax, regardless of the specific/unspecific divide (cf. also "Inverse Linking Constructions" in this handbook).

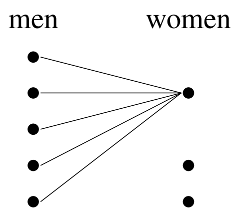
Example (9) therefore seems better suited than the often cited (1) to make a point for a real scopal ambiguity between an inverse and a linear reading. Unfortunately, the ∀∃R is unavailable for some speakers of languages other than English; in fact, most speakers of German do not get it (cf. Pafel (2006)). Moreover, even if the critical reading can be obtained, this does not settle the matter completely. As the reader may already have noticed, it still holds that one construal of (9) (namely the ∃∀R) logically implies the other. Hence, if we take the weaker construal as basic, we could still derive the stronger one as a special case. Of course this would presuppose that the inverse reading is the baseline, which seems much less plausible than taking the linear reading as a point of departure. In any case, whether or not the stronger reading is indeed a reading, we established that many speakers of English seem to get a reading where the subject is scope-dependent on the object, and this reading does not correspond to the linear syntactic structure that would interpret the subject before the object.

In order to avoid the problem of stronger and weaker "reading", one should consider logically independent readings: neither reading should be logically stronger (or weaker) than the other. If such cases can be found, we are on the safe side and can establish an indubitable ambiguity. Such examples can easily be found by slightly modifying the second QE of (1):

(10) Every man loves exactly one woman

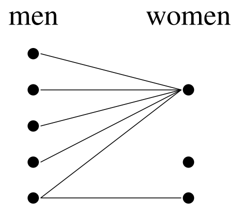
This time we find models for the wide scope reading of *exactly one* (abbreviated as !∃∀R) which are not at the same time models for the narrow scope reading (abbreviated as ∀!∃R) and vice versa. First consider (2) again which models the linear reading. An obvious candidate for the wide scope reading of *exactly one woman* would be (11):

(11)



However, although being a model for the !∃∀R, (11) would also verify the ∀!∃R; hence, this model cannot be used to bring home our point (recall that we have to find a model for which one reading is true but the other is false!). But now consider (12),

(12)



This model does not verify the ∀!∃R (the man at the bottom loves more than one woman) while still supporting the wide scope !∃∀R (there is no more than one woman being loved by all men).

What can we conclude from these examples? The argument goes as follows: Suppose you hear sentence (10) and you are asked whether the sentence is true in the model (12). Suppose your answer is "yes". Then this is possible only if you have the wide scope reading for the object in mind, because in this situation the narrow scope reading would be false. This proves the inverse reading to be real and indeed to constitute a "reading". Moreover, the linear reading is unproblematic; hence we have established two readings, and the argument is not flawed by entailment or specificity.

The argument can of course not be reproduced for sentence (1), but by analogy and uniformity it is assumed that the inverse ∃∀R for (1) is also "real". Hence there is reason to believe that our first analysis in terms of relative scope is still viable and moreover plausible enough, over and above an independently motivated distinction between specific and non-specific readings.

1.3 Next up on the agenda

Having interpreted scope and scope dependence in a rather intuitive informal way, we will now elaborate on the theory of quantification. The prevailing view to be found in all handbooks on logic and semantics since Stechow and Wunderlich (1991) is the **Theory of Generalized Quantifiers (GQT)**. In this theory, quantifiers denote relations between sets, much as predicates like *love* denote relations between individuals. This will be discussed and critically evaluated in section 2. The theory of Generalized Quantifiers goes along with a theory of LF to be sketched in section 3. Having done justice to the mainstream by discussing these theories I will argue that this theory tells us little about one's basic and perhaps naive intuitions about scope and quantification. Thus, the way I understand these notions is in terms of choosing an individual from a certain domain, namely the retriction of the quantifier, and using this individual as the semanic value for an argument slot occupied by the QE in syntax. The issue of scope dependency arises if this choice must depend on a further parameter, so that in different contexts a different individual should habe been chosen. Typically, such a context-dependent choice is induced by the interpretation of other linguistic material, namely a "scope inducing" expression (like universal quantification). I will elaborate on this view in section 4.

2 Generalized quantifiers

The motivation for analysing QE as relations between sets is grounded in the linguistic analysis pioneered by Montague and others in the late sixties of the last century. Simplifying a lot, one may assume that a sentence like *Peter loves Mary* can be analysed as saying that Peter is an element of the set of individuals that love Mary: Peter $\in \{x : x$ loves Mary$\}$. However, *every man loves Mary* cannot be so analysed, as *every man* does not denote an individual (for a formal argument that QEs cannot denote individuals or sets of individuals, cf. the “Postscript” p. 49ff in Peters and Westerståhl (2006)). Rather, it is assumed that, given the VP denotation $\{x : x$ loves Mary$\}$, the correct truth conditions can be derived only if the VP extension is an element of the denotation of *every man*. So what is the denotation of *every man*? Assume that *man* denotes the set $\{x : x$ is a man$\}$. If so, semantic intuition tells us that the denotation of *every man* must be the set of sets Z, so that the denotation of *man* is a subset of Z:

(13) *Every man loves Mary* is true iff
$\{y : y$ loves Mary$\} \in \{Z : \{x : x$ is a man$\} \subseteq Z\}$ iff
$\{x$: x is a man$\} \subseteq \{y : y$ loves Mary$\}$

This way, the statement that every man loves Mary reduces to asserting a relation between two sets: the denotation Y of the VP and the denotation X of the restriction of the quantifier *every*. But which relation? Obviously, this must be the subset relation, so that the meaning of *every* can be described as in (14):

(14) *every* = $\{\langle X, Y\rangle : X \subseteq Y\}$

The decisive observation, however, is that this seems to work for all other QEs in exactly the same way, e.g.:

(15) *no* = $\{\langle X, Y\rangle : X \cap Y = \emptyset\}$
some = $\{\langle X, Y\rangle : X \cap Y \neq \emptyset\}$
most = $\{\langle X, Y\rangle : card(X \cap Y) > card(X)/2\}$
five = $\{\langle X, Y\rangle : card(X \cap Y) \geq 5\}$
exactly five = $\{\langle X, Y\rangle : card(X \cap Y) = 5\}$
all but ten = $\{\langle X, Y\rangle : card(X - Y) = 10\}$
more than seven out of ten = $\{\langle X, Y\rangle : card(X \cap Y) > (7/10) \times card(X)\}$
neither = $\{\langle X, Y\rangle : card(X) = 2$ and $(X \cap Y) = \emptyset\}$
etc.

The only additional work to be done is to reconcile this relational view with compositional syntax. This is done by analogy to any other two-place relation, e.g. the relation *love* that takes an object to form a set, as explained in any textbook on formal semantics. By analogy, the quantifier takes as an object the denotation of its restriction to form a set of sets. As a result, all QEs denote set of sets. For example, as stated above in (13), the QE *every man* denotes the set of all supersets of the set of men ($\{Z : \{x : x$ is a man$\} \subseteq Z\}$), and the sentence *every man snores* is true iff the set of snoring entities is an element of that set. Likewise, the QE *no man* denotes the set of all sets whose intersection with the denotation of *man* is empty. To see the general pattern, consider (16):

(16) *every X* = $\{Y : X \subseteq Y\}$
no X = $\{Y : X \cap Y = \emptyset\}$
some X = $\{Y : X \cap Y \neq \emptyset\}$

most X = $\{Y : card(X \cap Y) > card(X)/2\}$
five X = $\{Y : card(X \cap Y) \geq 5\}$
exactly five X = $\{Y : card(X \cap Y) = 5\}$
etc.

Effectively, then, the interpretation of any natural language QE boils down to a statement about two sets: 1. the restriction X of the quantifier and 2. what is called the (syntactic) scope Y (which I will sometimes refer to as **scope**$_Q$). As this statement is about sets and its elements, it is by and of itself quantificational; e.g., the subset relation in (14) is expressed in the meta-language by saying that *every* element of X is an element of Y, the non-emptyness of an intersection is expressed by saying that there *exists* an element in both sets, etc.

However, this general format as a relation between sets (or a set of sets) does not characterize the concept of quantification: not any conceivable relation would intuitively qualify as a quantifier. And here is where the **Theory of Generalized Quantifiers** (GQT) enters the stage.

GQT starts off with the assumption that any such relation is a potential quantifier, but the theory then strives to single out the intuitively "real" quantifiers by axiomatically stating properties of these relations that any natural language quantifier must have. It is hoped, then, that the theory can narrow down potential quantifiers to only "actual" ones, i.e., ones that meet the axioms and can be expressed in natural language, cf. Barwise and Cooper (1981) or Keenan and Stavi (1986). Moreover, the theory states additional specific mathematical properties of certain *types* of quantifiers and thereby provides for a nice *classification* of different (natural) classes of quantifiers; see Keenan (2011) for a survey.

The theory and its mathematics has filled books (cf. e.g. van Benthem and ter Meulen (1984) or Peters and Westerståhl (2006) for extensive surveys.) and has widely been recognized as the standard theory for QE in linguistics. However, in recent years, some doubts came up as to the adequacy of GQT as a theory for natural language. Before discussing this criticism, let us briefly mention some of its merits.

Admittedly, some of the axioms characterizing quantifiers have intuitive appeal and help to clarify the concept of restricted quantification, although in a very abstract and indirect way. For example, the following property of NL-quantifiers, called **conservativity**, naturally suggests itself:

(17) For any quantifying relation R, $R(X, Y)$ iff $R(X, Y \cap X)$.

Applying this restriction to *every* and arbitrarily chosen X (e.g. *man*) and Y (e.g. *snore*), (17) correctly predicts that the following should be a tautology:

(18) every man snores iff every man is a man who snores

The reader is asked to check (17) for the quantifiers discussed above. As (17) is true in all these cases, the conjecture has been made that (17) is valid for all QE and indeed helps to characterize the concept of quantification in natural language. (17) also massively reduces the number of possible relations between sets. For example, given a universe of only 2 elements, there are potentially $2^{4^2} = 2^{16} = 65536$ different quantifiers, but given (18) this reduces to only $2^{3^2} = 2^9 = 512$ (cf. Keenan and Stavi (1986) or Thijsse

(1984)).

One of the advantages of Montague's analysis is that all DPs are uniformly considered sets of sets. That is, all DPs have a uniform logical type. This can even be extended to proper names. Above we already mentioned that in some theories, proper names are assimilated to quantifiers as regards their property of always having "wide scope" (or being scopeless). But how can we analyse names as quantifiers even if they are not quantificational in the intuitive sense? The answer is that proper names, although not qualifying as quantifiers in GQT, can be given the same format as genuine quantifying expressions, i.e. can be analysed as sets of sets. E.g. Montague analysed the denotation of *John* as the set of properties John has, which is the set $\{X : \text{John} \in X\})$ generated by John. Although such an analysis (which involves a "lifting" operation also known as **type-shifting**) may seem rather awkward, it has certain technical advantages for the syntax semantics interface: it makes for a uniform analysis of DPs and predication: E.g. *John snores* is analysed parallel to *every man snores* by saying that in both cases of subject-predicate constructions the sentence is true iff the predicate's denotation is an element of the denotation of the subject.

Let us call all denotions of DPs, i.e. all sets of sets, **Generalized Quantifiers (GQ)**. This includes also names, which do not qualify as genuine QEs. One of the advantages of GQT is that we can now prove that names are special in that they cannot induce scope. In fact, Zimmermann (1993) formally proved that names are the *only* scopeless GQs. This can be demonstrated by showing that in any model, for any GQ α, for any GQ β, and for any two place relation R, the (simplified) expressions $\beta_x \alpha_y R(x, y)$ and $\alpha_y \beta_x R(x, y)$ are logically equivalent if and only if β is generated by a proper name as described above (algebraically speaking, β must be an ultra-filter). This result crucially hinges on a unified analysis for both proper names and real QEs.

Let us now turn to the more disconcerting features of GQT.

First, it seems to me that the number of possible quantifiers left as a result of such axioms as (17) is still huge. This might perhaps be due to the fact that the theory, as it was originally conceived, embraces all sorts of expressions including boolean combinations like *less than 5 or more than 10* or *more than 5 but not exacly 10* which are obviously composed out of more primitive quantifiers. However, there is nothing in the theory that would allow for isolating exactly the "primitive" or even the "lexical" quantifiers of a natural language.

Second, a related a priori objection is based on the compositionality of complex quantifiers. E.g., the theory treats *more than 5* as a primitive quantifier, but of course one would like to decompose it into constituents that compositionally account for the contribution of the individual words; cf. "Modified Numerals" in this handbook. Moreover, even when considering simple numeric quantifiers as *fifty-six* there is a clear sense in which the numeral itself can be decomposed (50 + 6) and has a recursive compositional semantics of its own. And finally even seemingly primitive quantifiers like *more* can be decomposed semantically into *x-many* and a comparative morpheme (cf. Hackl (2009)).

Third, in the context of numerals the issue of *plural quantification* becomes a matter of importance. Consider predicates like *meet* which apply to sets of individuals. If so, it seems that plural nominals also apply to sets and that the most straightforward logical analysis of *Two men met* would be (19):

(19) There is a set X such that two(X) and men(X) and met(X)

In (19), the numeral *two* is analysed in a Fregean style as a property of sets, namely as the property that applies to any set that has (exactly) two elements, and the only quantifier in (19) is second order existence. If this is how it should be, the numeral would behave more like an adjective than like a genuine quantifier. In any case, the GQT must somehow be modified in order to deal with pluralities; we will come back to the problem of plural predicates and the relation between first and second order quantification in GQT below in section 6 on distributivity.

Fourth, and as a further consequence of lack of decomposition, it has been claimed that the theory fails on a purely descriptive level. E.g., *more than n* and *at least n+1* have equivalent semantics in GQT, but can be shown to imply different truth conditions. According to Geurts and Nouwen (2007), the difference is due to a modal epistemic component in the meaning of the latter quantifier that is lacking in the meaning of the former. The question then arises whether this is really a difference in core meaning, or a difference on the level of implicature.

Fifth, the theory has nothing to offer as an account for scopal variance between GQs. E.g., the bare numeral in (20a) allows for scope reversal, but not so the modified numeral in (20b):

(20) a. Every student read five papers
b. Every student read more than five papers

A possible explanation might result from the decomposition into a comparative unit which cannot take scope over the subject, cf. Takahashi (2006).

Sixth, I would like to express doubts on the usefulness of classifications that result from formal properties of quantifiers in GQT. The most popular example is the property of decreasing monotonicity. A GQ is **decreasing** (or **downwards entailing** or **downwards monotone**) iff for any X, Y, if $X \subseteq Y$ and $Y \in GQ$, then $X \in GQ$. Example: The quantifiers *no* and *few* are decreasing. Let X be the things that walk and talk, let Y be the things that walk, then $X \subseteq Y$ and if no fish walks, then no fish walks and talks. Now, the hope of GQT is that these formal properties, here: the classification as up- or down-ward entailing, is directly reflected in linguistic behavior that can only be explained by recourse to these subclasses of GQs.

The usefulness of downward monotonicity is generally demonstrated by showing that among the GQs it is precisely the decreasing quantifiers that license negative polarity items (NPIs) like *ever*, as in *no/few fish ever walks*. However, downward entailment is not a property specific to quantifiers, as simple negation also licenses NPIs. What is more, it has frequently been argued that negative quantifiers must be decomposed into negation and existential quantification, as in the so-called Neg-Split reading illustrated in (21):

(21) The company need fire no employees
= It is not the case that the company is obliged to fire an employee

Likewise, some sort of decomposition is also required in the analysis of negative concord, cf. "Negative Indefinites and negative concord" in this handbook. Generalizing a bit, downward entailing QE are not per se downward entailing in LF; rather they decompose into an existential quantifier occuring in a negative environment. But if negation as such is present at LF, the appeal to downward entailing quantifiers has become spurious, as the effect is simply caused by the presence of negation.

Seventh another consequence of decomposition seems to affect seemingly QE like *only John* or *only dogs*. First note that (22a) and (22b) are not logically equivalent; hence *only* cannot be conservative and therefore, according to the axioms of GQT, does not qualify as a natural language quantifier.

(22) a. Only dogs bark
b. Only dogs are dogs that bark

Moreover, *only dogs* should not be analysed as a constituent in LF, akin to negation in (23):

(23) Not every boy can be above average height

The paraphrase in (24) shows that *can* must have wide scope over the universal quantifier, but must still be in the scope of *not*, suggesting that *not* and *every boy* do not form a constituent:

(24) It is *not* possible that *every boy* is above average height

Likewise, it has been argued for German (cf. Büring and Hartmann (2001)) that (25a) and (25b) are not logically equivalent:

(25) a. Nur seine Mutter liebt jeder
'only his mother everyone loves'
b. Jeder liebt nur seine Mutter
'everyony loves only his mother'

(25b) says that for every person x there is only one other person being loved by x, namely x's mother. (25a) does not exclude the possibility that x loves more than one person. It only says that among them there must be x's mother. A proper analysis for (25a) seems to involve a split between *nur* (only) and *seine Mutter* (his mother) along the lines of (26),

(26) Nur [jeder liebt [seine Mutter $]_F$]
only everyone loves his mother

where the pronoun *his* is still bound by *everyone*, but *only* operates as a kind of focussing operator. By analogy, *only dogs* can therefore not form a constituent at LF, and therefore cannot be analysed alongside with other bona fide QEs. Still, the result that *only dogs* is not a QE is intuitively somewhat irritating, as the procedure to falsify *Only dogs bark* is clearly in a sense "quantificational": all one has to do is (choose and) find a barking species that is not dogs (e.g. foxes).

Eighth GQs are unlike quantifiers in logic as they are unable to bind variables. This is obvious from the logical type of quantifiers as relations between sets and that of GQ as sets of sets. Perhaps it should be noted that the term Generalized Quantifiers when first showing up in the literature was still meant as an extension of the classical quantifiers $\exists$ and $\forall$ in logic (with the important result, that quantifiers like *most* cannot be treated in classical first oder logic, cf. Mostowski (1957) or Lindström (1966)) and were variable binders, but the theory lost this property when adapted and modified in linguistics. As a result, there is no standard surface analysis for sentences like

(27) Every boy loves his mother

with the reading where *his* is anaphoric to *every boy*. Obviously, *his* should be interpreted as a bound variable, but cannot be so since *every boy* is not a binder, as would be the case for quantifiers like $\forall$ and $\exists$ in logic. On this, Keenan (2011) in his handbook article comments: "But quantification and variable binding are different operations, so it is semantically enlightening to separate them" (p. 1062). However, no empirical or conceptual argument is given for this separation, and without further justification I am not convinced that the lack of the binding capacity is actually a virtue of GQs. We will see in the next section, how GQT solves this problem.

Summing up so far, we argued that GQT has its problems, although it provides for a uniform semantics for all types of DPs. We have shown above that this uniformity cannot do justice to lexical decomposition. In fact, seen from this perspective, the "logical" quantifiers drastically reduce to only a handful: existential quantification over individuals, existential quantification over sets (as in (19): "there is a set X ..."), universal quantification over individuals, and perhaps some specific second order QEs that resist lexical decomposition, as has traditionally been assumed for the proportional quantifiers *most*. But even in this case there are proposals that make *most* part of the restriction of a second order quantifier, cf. Hackl (2009).

It thus seems that the overwhelming majority of what is called QEs can be decomposed into (1.) "logical" quantification as described above, (2.) "quantitative", i.e. cardinal properties prototypically expressed by numerals which go into the restriction of the second order existential quantifier (3.) possibly additional pragmatic or modal components, e.g. (3a.) an implicit contextually determined standard of comparison, as for *many* and *few*, (3b.) an epistemic possibility as in *at least n* (= *possibly more than n*), and/or (3c.) a quantitative implicature, as for *some*, *few*, and *most* (= not *all*). This suggests that the lexical content of many QEs basically reduces to its NP and a cardinal property, expressed as part of the restriction, but the "real" (logical) quantifier might not be part of the lexical meaning of the QE but is present at LF only as something that is tacitly inserted, following the pattern of (19) and that of bare plural DPs which lack an overt determiner but nonetheless require this (covert) determiner to be semantically contentful.

In consequence, what many of the axioms of GQT in fact do is characterize a property of (part of) the restriction, namely what we referred to above as its cardinality. E.g., Mostowski and Lindström characterized GQs as "topic-neutral" in the following sense: If the individuals in a model are mapped one-to-one onto another set of individuals, the truth conditions of a GQ must be preserved. This axiom, called **isomorphism**, precisely characterizes QE dealing only with quantities. This is an aspect of QE we identified above as being lexically expressed by a numerical expression as part of the restriction in (2.) above. That numerals have this property of dealing with quantities doesn't come as a surprise. It seems, therefore, that the axioms of GQT cannot be primitives, but should be derivable from a more fine-grained theory of natural language expressions.

At the end of the day, then, it might well turn out that the only primitive quantifiers are just the ones mentioned in (1.) above, namely universal quantification and existential quantification over individuals and sets. If correct, GQT did not offer much help for the linguistic analysis of quantification. Nonetheless, and much to my regret, GQT has become the standard tool in the field. So it will be necessary to do justice to this tradition; the standard theory may serve as a reference point against which other theories (not based on GQs) can be evaluated. We follow textbook wisdom as expounded in Heim and Kratzer (1998).

3 Logical Form

Let us first start with a discussion of GQT's solution to the challenge of bound variable pronouns. It will be instructive to see how this problem is solved, as the solution turns out to be applicable to problems of quantifier scope as well.

3.1 Bound variable pronouns

In this section we describe the strategy taken by GQT towards binding (as separate from quantification). Consider a logical representation of (27) as one would expect in the tradition of Montague Grammar.

(28) $\forall x(\textbf{boy'}(x) \rightarrow \textbf{love'}(x, \textbf{mother-of'}(x)))$

The variable x here occurs both as the translation of *his* and as the subject of *love*, being bound by the same quantifier. However, this format is a far cry from natural language syntax; by the denominational lines of GQT, the QE cannot bind but instead has to relate two properties, corresponding to the restriction and the scope$_Q$ of *every*. The restriction is unproblematic; this is simply the predicate *boy*, or in set theoretic notation $\{x : x$ is a boy$\}$. And for the scope$_Q$ it seems straightforward to assume that this is $\{x : x$ loves his mother$\}$. In addition we derive $\{x : x$ loves x's mother$\}$ by identifying *his* with the variable x.

In order to generate this property, a syntactic transformation (called **quantifier raising**, or QR for short) is postulated that does a number of different things. Starting with (27), the first step is to move the QE into a position where it is adjoined to a sentence, thereby leaving behind a trace t:

(29) Every boy [t loves his mother]

As usual, the trace and its antecedent are coindexed, say by 1:

(30) [Every boy]$_1$ [t$_1$ loves his mother]

Moreover, the anaphoric reading of *his* is represented by co-indexation:

(31) [Every boy]$_1$ [t$_1$ loves his$_1$ mother]

The reason for this is simple: the logical formula (28) reveals that the subject must be interpreted by the same variable as the pronoun *his*. We therefore must assume that the trace and the coindexed pronoun get interpreted by the same variable; e.g. x_1.

The next step in this process is a certain re-encoding of the information contained in (31) by removing the index of the QE and attaching it to an operator adjoined to the sentence as shown in (32):

(32) [Every boy] [Op_1 [t_1 loves his_1 mother]]

This representation is the **Logical Form** of (27) and thus serves as the input to semantic interpretation, according to the following rules:

First, if we want to derive the bound variable reading, it is essential that the pronoun and the trace corefer. This is guaranteed by assuming that pronouns and traces are interpeted by the same variable if and only if they are coindexed:

(33) [Every boy] Op_1 [x_1 loves x_1's mother]

Second, we must make sure that the operator Op_1 is interpreted as set formation, using the index on the operator as the set forming variable, as shown in (34):

(34) [Every boy] {x_1 : x_1 loves x_1's mother}

The set so-formed is the desired interpretation of the scope_Q of *every*. Again, coindexation is crucial and comes along with QR.

Finally, the resulting configuration is semantically interpreted by the subject-predicate rule already mentioned, it says that for the sentence to be true the set formed by the operator must be an element of the subject (the GQ). Thus, *every boy loves his mother* is true iff the set of boys is a subset of the set of x_1 such that x_1 loves x_1's mother. QED.

This derives the correct truth conditions, but at a certain price. We depart from the syntactic surface structure by resorting to an abstract level of LF. This representation contains two essentials to make the theory work, namely the binding operator Op (set formation, or lambda abstraction) and the use of indices to select the correct variables. Thereby, the paradox that GQ are not binders is solved by postulating that *all* GQs move at LF and that movement of a DP always creates a binder (the above operator Op_i). This latter assumption is as crucial as it seems ad hoc. Nonetheless, this theory has become the silver bullet, in fact a purportedly ideal solution, to a number of other problems I will discuss below.

3.2 Type mismatch

One of these problems for compositional semantics is QE in object position, as illustrated already in (1) or even simpler constructions like (35):

(35) John climbed every tree

Given that the verb *climb* denotes a relation between *individuals*, there is no obvious way to combine this relation with a set of sets (the denotation of the GQ *every tree*). What we have here is a so-called "**type mismatch**" that necessitates some kind of operation which removes the conflict between logical types. Now, one of the solutions to this problem is movement, as it may leave behind a trace that can be interpreted as a variable of the right kind.

(36) LF of (35): [Every tree] Op_i [John climbed t_i]

The mismatch is now removed by interpreting the trace t_i as a variable for an individual. The semantics of (36) is straightforward: Op_i [John climbed t_i] is interpreted as $\{x : x$ John climbed $x\}$. This in turn is an element of *every tree* if and only if (35) is true.

3.3 Scope

We will now analyze more closely how ambiguities of the kind discussed at the beginning of our article may be treated in the framework of GQs. Recall that many speakers of English find sentences such as (37) ambiguous, thus allowing for a linear (though sometimes implausible) and an inversed reading with the object having wide scope with respect to the subject.

(37) a. A table touches every wall
b. A boy climbed every tree
c. A rose filled every vase
d. A flower graced every table

How can we get the truth conditions of such sentences correct in a rule governed way? Here again, QR comes in handy. First consider QR of the object:

(38) every wall [Op_1 [a table touches t_1]]

The semantic effect is the formation of a property $\{x_1$: a table touches $x_1\}$. Clearly the choice of a table will now depend on the choice of x_1; this way it is clear that *a table* is interpreted as scope dependent on *every wall*. Let us look finally at the constituent *a table touches* t_1. Assuming that *touch* is a verb that takes an individual as an argument, rather than a GQ, we have to apply QR again, yielding the LF in (39):

(39) every wall [Op_1 [a table [Op_2 [t_2 touches t_1]]]]

Observe that the indices of *every wall* and *a table* must differ. Thus, indices are a crucial part of the semantics, over and above the issue of bound pronouns.

The attentive reader might ask how to get the implausible reading with a single table touching every wall. This can be done by simply changing the order of the QR operations:

(40) a. a table [Op_1 [t_1 touches every wall]]
b. every wall [Op_2 [a table [Op_1 [t_1 touches t_2]]]]

Likewise, the ambiguity of (1) and related examples can simply be accounted for by assuming different orders of rule application in the process of generating LFs. As a consequence, the resulting LFs will exhibit different scope relations that account for the semantic ambiguity. Thus, apart from solving the binding problem and removing the type mismatch, QR also accounts for ambiguities.

A fourth motivation for QR and LF, although an indirect methodological one, relates to a very general principle that is supposed to govern the syntax-semantics interface. As can easily be verified the above analyses are in accord with the following **Scope Principle**:

(41) If α is in the semantic scope of β, then β must c-command α.

This principle is naturally satisfied by surface syntax in the $\forall\exists$R of (1), as the subject always c-commands the object. In the inverse reading of (1), however, it would be necessary for the object to c-command the subject which is possible only if there is some movement going on at the level of LF.

There are two further applications of the theory, both of which I do not have the space to discuss here. One is an analysis of so-called antecendent-contained deletion, the other an analysis of so-called sloppy readings in elliptical constructions. I have to refer the reader to the textbook of Heim and Kratzer. As concerns ellipsis, cf. Dalrymple *et al.* (1991) for an an alternative to QR in elliptical sloppy identity constructions. With respect to antecedent-contained deletion, Hackl *et al.* (2012) discuss experimental data including reading times and off-line ratings of sentences in order to decide between two competing types of theories on quantifier interpretation, namely QR and type shifting theories (which will be briefly mentioned below in the context of Categorial Grammar). They claim that the data are compatible with QR but inconsistent with type-shifting theories. In response, Szabolcsi (2014) claims that, under a broader conception of type-shifting theories, Hackl et al.'s (2012) original data are actually just as compatible with type-shifting as they are with QR. In addition, Gibson *et al.* (2015) argued that Hackl et al. made processing assumptions with regard to the QR theory that are implausibly strong. According to Gibson et al., the experimental findings are in fact inconsistent with the QR theory, but can be accounted for by the type shifting theory using yet other processing assumptions.

3.4 Restrictions on QR

As shown in the last section, QR is a very powerful tool for analysing diverse phenomena in a uniform way. On the other side of the coin the method seems too strong, as it allows for various sorts of overgeneration to be discussed in this section.

3.4.1 Binding

A case in point is handled by the following **Binding Principle** that imposes a constraint on the relation between surface structure (before QR) and LF:

(42) If a QE binds a pronoun at LF, it c-commands that pronoun before QRing of the QE.

This restriction (cf. the "'Binding Principle" in Heim and Kratzer (1998, p. 264ff)) intends to rule out a reading of

(43) Her_1 friend loves every girl_i

meaning “every girl is loved by her friend” which would become possible in a LF like this:

(44) every girl [OP_1 [her_1 friend loves t_1]

Here, the universal QE (indirectly) binds *her*, which is ruled out by (43) because *her* is not syntactically bound (= c-commanded by its antecedent) in (43). It follows that the indexation in (43) is ill-formed, because (obligatory) QR violates (42). The principle therefore accounts for all cases in which a QE has been moved across a coindexed pronoun. Such cases are also known as “crossover” or superiority violations, cf. e.g. Postal (1971, 1993), Kuno (1988), Lasnik and Stowell (1991), or Safir (1996) for syntactic analyses or Barker and Shan (2006) as an attempt to derive these effects by applying constraints on semantic processing.

3.4.2 Clause-boundedness

It is generally assumed (cf. May (1977, 1985)) that QR is syntactically restricted in such away that the raised quantifier cannot gain scope beyond the clause from which it originates. In other words: QR is clause-bounded. Consider the following examples:

(45) a. A boy climbed every tree
b. the boy’s claim [to have climbed every tree]
c. A boy claimed [to have climbed every tree]
d. A boy claimed [that Peter has climbed every tree]

Many speakers accept an ∀∃R for (45a), but few if any accept the ∃∀R for (45b,c,d). By contrast, examples like

(46) a. every boy has climbed a tree
b. every boy claimed to have climbed a tree
c. every boy claimed that many girls have climbed a tree

exhibit a ∃∀R, even in cases like (46b,c) where the wide scope of *a tree* contradicts clause-boundedness. This difference between the universal and the indefinite quantifier can be explained by assuming that *a tree* can have an independent specific reading. Since such a reading does not involve QR, (46b,c) would not require long movement so that we can stay consistent with the above generalization.

However, there seem to exist real counterexamples to clause-boundedness. Consider (47a):

(47) a. At least one referee recommended that we should accept every paper
b. At least one referee recommended that every paper should be accepted

It seems that *every paper* can gain scope across *at least one* in (47a), but not in (47b). Assume that QR is not clause-bound. The apparent lack of wide scope in (47b) could then be explained by an additional principle, called the “empty category principle” in Chomsky (1981), which blocks long movement from subject positions. But still it remains unclear how to block scope inversion in (45).

Returning to the specific reading of indefinites, given that such indefinites are always scopeless, they hence generate the illusion of widest scope. This would explain that indefinites behave as if they could escape the purported clause-boundedness restriction of QR. But now consider (46c) again: following Fodor and Sag (1982) we assume that there is a reading that could be paraphrased as in (48):

(48) For every boy x there is a (possibly different) tree such that x claims that many girls have climbed that tree

According to this reading *a tree* has wide scope outside its clause, but narrow scope with respect to the matrix subject. This kind of scope-dependence is impossible if the indefinite DP is specific, hence scopeless. Likewise, consider the possible scope relations of *some condition* in (49) from Farkas (1981), cf. also Abusch (1994):

(49) Each student has to come up with three arguments which show that some condition proposed by Chomsky is wrong

It's not implausible to interpret *some condition* as scope independent on *three arguments* but still scope dependent on *each student*, thus ruling out the interpretation of *some condition* as specific (= equivalent to outmost wide scope).

The gist of the discussion seems to be that we either have to exempt indefinite quantifiers from clause-boundedness or find another non-standard way to interpret them. The latter type of theory is exemplified in Kratzer (1998) and Reinhart (1997) who argue that the existential is not a real quantifier but nevertheless can exhibit a scope dependence. This latter type of dependence is called **pseudo-scope** and various theories have been proposed how this kind of dependency can arise. We will discuss one such proposal in section 4.1.

3.4.3 Clause-internal restrictions

Since the early days of natural language semantics it has been observed that sentences like

(50) a. Many men read few books
b. Few books are read by many men

(51) a. I talked to few girls about only those problems
b. I talked about only those problems to few girls

are not equivalent, as this particular combination of quantifiers does not allow for scope reversal, "at least for the majority of English Speakers" (Lakoff (1971, p. 240)).

This potential overgeneration of QR by scope reversal is a persistent problem for any theory of quantifier scope: what are the restrictions operative in such data? E.g., we already observed in (20b) that complex quantifiers are less likely to take wide scope. Other examples suggest that this restriction is in need of qualification, as *more than one journal* in (52) can have a wide scope reading, but *more than one paper* cannot.

(52) a. Every student read more than one paper
b. John submitted every paper to more than one journal

Moreover, sentence negation seems to block wide scope of the object in (53):

(53) John didn't read every book

However, this restriction cannot be purely syntactic, as the wide scope reading can be regained if the subject is an indefinite:

(54) A boy didn't read every book

It seems that wide scope is possible only if it induces scope depencence, which is not the case in (53). See Beghelli and Stowell (1997) for more data and more sophisticated explanations.

Other data suggest that scope differences can subtly depend on the semantic role of the QE; e.g. Lakoff (1970, p. 405) observes that *many children* can gain wide scope in (55a) but not in (55b):

(55) a. Fathers of many children read few books
b. Fathers with many children read few books

Finally, investigations of German mostly concentrate on the possible effects of reconstruction, i.e. the undoing of movement, as in (56a), as opposed to (56b):

(56) a. Jeden dieser Schüler$_{acc}$ lobte genau ein Lehrer$_{nom}$ t$_{acc}$
each of-these students praised exactly one teacher
'Each of these students, exactly one teacher praised (him)'
b. weil genau ein Lehrer$_{nom}$ jeden dieser Schüler$_{acc}$ lobte
because exactly one teacher$_{nom}$ each of-these pupils$_{acc}$ praised
'because exactly one teacher praised every student'

As shown in Bott and Schlotterbeck (2015), (56a) is ambiguous, due to the possibility of reconstruction, whereas there is no movement and no reconstruction in (56b) which has unambiguous linear scope.

3.5 Discussion

Summarizing so far, the clause-boundedness condition is both too strong and too weak. Rather, scopal interactions depend on

a. the kind of quantifier that wants to gain scope (bare numerals vs. modified numerals vs. negative quantifiers vs. comparative quantifiers vs. distributive universals vs. collective universals like *all* etc.);
b. the kind of syntactic construction, e.g. reconstruction or the "**inverse linking** construction" (cf. the article on "Inverse Linking Constructions: An apple in every basket" in this handbook), or on the influence of grammatical functions like subject, indirect object, direct object, cf. Ioup (1975);
c. the kind of semantic role of the quantifier (topic, agent, experiencer, theme), cf. Kurtzman and MacDonald (1993);
d. the kind of semantic effect on other quantifiers, as evidenced by (54) (cf. also Fox (1995));

e. various island conditions as illustrated by (45) and (47);

f. the influence of the context in disambiguation (cf. Anderson (2004)),

g. the kind of language under investigation with respect to different scopal possibilities (cf. e.g. Liu (1990) for Chinese and Pafel (2006) or Bott and Schlotterbeck (2012, 2015) for German) and different means to express quantification (cf. Matthewson (2008) or Bach *et al.* (1995)).

The combinatorics of these factors is exploding and the literature on the subject is vast (as a focussed point of reference, see the articles collected in Szabolcsi (1997)). To date, there is no satisfying theory that could come close to a uniform explanation doing justice to a sufficiently significant amount of data. This may be due to the fact that the phenomenon under investigation is heterogeneous; there is no single grammatical module that characterizes scope. Rather scope seems to be the result of complicated interactions of the above mentioned factors.

Contributing to this diversity there are a range of different semantic theories that present themselves as alternatives to QR. We have seen above that QR, although much too strong in its unrestricted form and at the same time too weak, derives its attractiveness from its simplicity and the simple semantics it presupposes. But it simplifies semantics at the price of complicating the syntax by adding an otherwise unmotivated level of LF and an even less motivated surface syntax. As a case in point, consider the analysis of (57) in the Heim/Kratzer textbook (p. 228ff).

(57) No owner of an espresso machine drinks tea

By QRing *an espresso machine*, we get wide scope over *no*, which is unintended. Hence we have to move *no owner-of t* further up across an espresso machine, but this is impossible since the espresso machine is an argument of *owner of*. The solution proposed by Heim and Kratzer is that *owner* has an empty subject position *e*, as shown in (58):

(58) an espresso machine Op_1 [*e* owner of t_1]

This empty position is then QRed which yields:

(59) *e* Op_2 [an espresso machine Op_1 [t_2 owner of t_1]]

(59) can then be interpreted as the set $\{x_2$: there is an espresso machine x_1 and x_2 owns $x_1\}$. This property then serves as the restriction of *no*, which yields the correct truth conditions. This only works if we assume an additional subject position otherwise unmotivated in syntax, and as pointed out by Heim and Kratzer, such positions must be postulated for "all lexically headed XPs" (p. 228).

Another discontent with standard theories is conceptual: If compositionality of semantic interpretation is an issue, it has become almost void in a theory of LF: Although LF itself can (hopefully, see below) be interpreted in a compositional way, the transformations generating LF cannot. That is, there is no straightforward semantic operation that corresponds to QR in a compositional way, simply because the input of such an operation is not interpretable. It might be for this reason that the transition to LF is claimed to be part of syntax, but in practice, it is used by semanticists as an ad hoc device and often as a wild card to get their logical representations straight. This considerably weakens the concept of compositionality.

Moreover, and even worse, LF is usually not interpreted in a strictly compositonal way. First, it should be noted that the set forming operator Op_i does not have a semantic value on its own, it must be interpreted "syncategorematically" as is the case for $\exists x_i$ and $\forall x_i$ in standard predicate logic, which can only be interpreted together with their scope. In the standard theory of predicate logic, however, this interpretation is not even compositional, as the semantic value of α in $\exists x_i \alpha$ must be modified as the result of quantification. The same applies to set formation (or lambda abstraction).

There are several reactions to this problem. The most popular is to ignore it. A radical solution to the problems that arise with the use of variables is simply to dispense with variables altogether, cf. Jacobson (1999) who proposes a variable free Categorial Grammar. Categorial Grammars were developed independently—and as systems still with variables—as a means to stay more close to the surface syntax and as an attempt to avoid QR as a general mechanism that is needed to remove mismatches. In fact there is no need in this theory to remove type mismatch by movement; rather predicates are "type shifted" in semantics so as to comply with the semantic type of GQs.

A simple version of type shifting is also present in Montague's treatment of transitive verbs. Here, the translation of *love* is not a relation between individuals, but one between individuals and GQs. Hence, QE in object position can be interpreted directly, without movement. In order to conform to the intuition that *love* ultimately does denote a relation between individuals, Montague assumes a meaning postulate:

(60) love(x,QE) iff QE({y: love'(x,y)})

Here *love'* is the underlying predicate, and *love* is its type-shifted variant. As the format of (60) reveals, there is still a kind of QR, but it's performed in the semantics, not in syntax.

The next step is to abolish QR as a means to gain scope. Hence, Categorial Grammars do not allow for a level of LF distinct from surface syntax (cf. Hendriks (1993), Steedman (1996), Jäger (2005), Morrill (2010), Moot and Retore (2012), Steedman (2012). The gist of these proposals is that inverse interpretations are a matter of semantic rules that combine constituents in inverse order. For example, we assumed above, following Montague (1973), that the VP is an element of the subject GQ, which generates linear scope. However, if the VP is type-shifted as in Montague (1970b), the GQ is an argument of the VP, which can generate inverse scope. As this scope reversal is rule governed, it might explain why it is relatively local, although the more fine grained details still remain mysterious. This notwithstanding, it also seems possible to explain that at least some unwarranted scope combinations of GQs can be ruled out on principled grounds, cf. e.g. Barker (2002).

The price to pay here however, is high, as Categorial Grammars exhibit a much more complicated system of logical types and interpretative processes like type shifting. And the price becomes even higher in a system without variables, where the complexity of logical types depends on the number of former "variables" each of which requires a global type shifting operation whenever a "variable" has to be explained away.

Moreover, the proposed solutions to avoid QR seem to necessitate different techniques for the different phenomena previously all covered by QR. Whereas QR seems to provide for a uniform (but incomplete) solution for a number of problems, other theories treat them in piecemeal fashion. E.g., the type shifting of predicates that resolves the type mismatch does not in and by itself lend itself to a mechanism that would allow for scope taking over other arguments. And the type shifting of predicates that allows for scope reversal does not in and by itself lend itself to a mechanism that would allow for extended binding possibilities. Whether this is good or bad may be disputed, the least thing we can say is that the exploration of alternatives to QR is an exiting area of research. But also a very difficult one. Recall all the different parameters for scopal interaction mentioned at the beginning of the section, the many different explanations and theories, and above all the uncertainty of intuitive judgments and disagreement between linguists in this area. It seems to me that current reseach has turned away from matters of scope in favor of problems that deal with the processing and the internal composition of quantifiers.

Finally, let me point out that the usual reaction to the problem of non-compositionality of Predicate Logic is to say that there is no problem at all because everything becomes compositional as soon as we can take variable assignments into account. It is true that in that case we can give a semantic value to expressions like $\exists x$, see e.g. Chapter 10 in Zimmermann and Sternefeld (2013). But as pointed out there, we again have to pay a price: if variable assignment functions become semantic objects, variables and indices themselves become part of our ontology. I guess this is not really what we want. Personally, I think that these problems are deep and worrying; in fact the use of variables is not as innocent as one might believe. The linguist usually does not care, but anyone interested in such philosophical problems might consult Klein and Sternefeld (2017).

4 Games as a (metaphoric) characterization of quantification

In this section I will briefly discuss an alternative conception of quantification, taking up the intuition already mentioned at the end of section 1.3 where I approached the notion of quantification by saying that its semantics always involves some choice of individuals or sets of individuals in a certain domain determined by the restriction of the quantifier. This conception will also help to elucidate the notion of scope and scope-(in)dependence.

I will make use of game theory (cf. Saarinen (1979), Hintikka and Sandu (1997)) as a metaphorical approach to quantification. My motivation for this choice is threefold: First, I strive to avoid a persistent problem when it comes to explaining primitive semantic concepts in logic: for example, when trying to explain the meaning of "and" it is almost inevitable to use the word "and" in the meta-language, and the same applies to bona fide quantifiers like *every*, *for all*, or *some*. To some extent, the metaphors of game theory allows us to avoid this impression of circularity. Second, I intend to characterize the purely logical effect of quantifiers in a very general manner; the specific effects pertaining to cardinality will not be part of this concept, as suggested above in the context of quantity, which seems to be a property of the restriction rather than the quantifier as such. Third, and most importantly, I want to analyze the concept of scope-independence by reducing it to a certain lack of information with respect to other quantifiers; the metaphoric language of game theory is particularly apt for explaining this concept in an intuitive way. A formal implementation can be given completely independently of the game theoretical packaging by the use of **Skolem functions**, see section 4.1. In this section we closely follow Hintikka (1979) and his exposition of the basic ideas; the only change we tacitly perform concerns logical notation.

Game theoretical semantics of First Order Predicate Logic presupposes a model M with a domain D in which the predicates of the formal language L of Predicate Logic are interpreted in the usual way. The aim is to determine the truth value of complex sentences. Each sentence or formula S is interpreted as a game between two players. We quote from Hintikka (1979, p. 34ff):

> [Each game G(S)] may be thought of as an idealized process of verification in which one of the two players, called 'Myself' or 'I', is trying to show that S is true, and his opponent, who is called 'Nature' and who is perhaps best thought of as as a Cartesian *malin genie*, is trying to show that it is false. My purpose in the game G(S) is to produce a true atomic sentence. If that happens, I have won and Nature has lost. If the game ends with a false atomic sentence, I have lost and Nature has won. Since the verification of an existential statement requires the searching for and the finding of a suitable individual, these verificational games are essentially games of seeking and (hopefully) finding.
>
> [...] what happens at a certain stage of the game is determined by the form of [a sentence] S'. We can distinguish the following cases depending on what this form is.
>
> G.E — If S' is $\exists x F(x)$, I choose a member of D, give it a proper name (if it does not have one already that can be used), say 'b'. The game is then continued with respect to $F(b)$.
>
> G.U — If S' is $\forall x F(x)$, the same happens except that Nature chooses b.
>
> G.or — If S' is (F or G), I choose F or G, and the game is continued with respect to it.
>
> G.and — If S' is (F and G), the same happens except that Nature makes the choice.
>
> G.not — If S' is 'not F', the roles of the players (as defined by the rules (G.E), (G.U), (G.or), (G.and), (G.not), and (G.A)) are reversed and the game is continued with respect to F.
>
> In a finite number of moves an atomic sentence A is reached containing only predicates of L and names of members of D. Winning and losing are defined with respect to it.
>
> G.A — If A is true, I have won and Nature lost; if A is false, vice versa...
>
> G.T — S is true if and only if I have a winning strategy in G(S).
>
> Here, 'strategy' is to be understood in the precise sense of the mathematical theory of games. [footnote: Cf. Luce an Raiffa, *Games and Decisions* (John

Wiley, New York, 1957).] The idea it embodies is nevertheless so natural that it can be understood without any familiarity with game-theoretical results or conceptualzations. A player has a winning strategy if he can choose his moves in such a way that, no matter what his opponent does, he in the end wins the game. (Of course his choices will in general depend on the opponent's earlier moves.)

It is most easily seen that if S is indeed true in the traditional sense, I can make my moves so that all the sentences S produced during the game are (apart from switches of roles induced by (G.not)) true in the traditional sense. Since this includes the outcome, I have a winning strategy. Conversely, if I have a winning strategy G(S), it is easily seen that S is true in the traditional sense. Hence what (G.T) defines is indeed equivalent with the traditional concept of truth.

Note that a single counterexample falsifies a universally quantified statement, and a single instance suffices to verify an existentially quantified one. Accordingly, the choice is made by Nature, the Falsifier, in case of universal quantification, whereas Myself, the Verifier, chooses an individual for an existential quantifier. The individual so chosen is interpreted as the value for the variable bound by the quantifier. Of course, instead of replacing the variable x by a name possibly added to the language, we could also follow standard practice by assuming assignment functions for variables.

Moreover, for restricted universal quantification we can slightly modify (G.U) in the obvious way. Let us work through our example (1). As the sentence begins with a universal quantifier (and assuming left to right processing), Nature has to choose an individual, and by restricted quantification, this individual must be a man. If there are no men in the model, Nature is unable to proceed and has to give up, so I have won (empty universal quantification is always true in logic). If there are men, Nature's strategy must be to choose a man for which Myself has no winning strategy. Of course Myself is wrecked if and only if it is impossible for Me to find a woman loved by the man chosen by Nature. If there is such a man in the model, the sentence is false (Nature has won), otherwise it is true (Me has won). The reader should check that these conditions correctly capture the truth conditions of (1).

4.1 Scope and pseudo-scope

Let us now discuss the notion of scope. Clearly, proper names are scopeless, as they do not depend on a selection procedure in the sense explained above. Existential quantifiers can be scope dependent, as My choice of an individual may well depend on the choice by Nature in a previous move of the game. And conversely, universal quantifiers can be scope inducing, as they make the choice in later moves depend on the choice made for the universal quantifier. To be more precise, the interpretation of Q_2 is scope dependent on Q_1 (in a certain formula) iff there is a model such that My choice of an individual for Q_2 in a winning strategy depends on that of My opponent, i.e. if My opponent had chosen a different individual for Q_2 than he actually did, I would also be forced to choose an individual different from the one I actually did.

Note that My choice is still restricted by the requirement that it is part of My winning strategy. So, in the case of (1) assume the linear reading and the model (11) which also satisfies the inverse reading. In that case My choice cannot depend on Natures choice, as there is only one woman that is loved (by every man) and that is My (only) winning candidate. Hence there is no dependence on previous choices *in that model*. Nonetheless we would like to say that, in principle, the existential quantifier can receive a scope dependent interpretation, hence we require in our definition above that *there is a model* that exhibits such a dependency. We must therefore abstract away from peculiarities of a given model.

Now consider the linear construal of *a man loves every woman*. Could Nature's choice depend on Mine? Well, for the sentence to be true, I have to choose a loving man so that the choice of a woman is immaterial. If I were to choose another man, the same remains to be true. If I were to choose a man who does not love all women, then Nature will win, but in that case I did not conform to the condition that My choice must be one that might constitute a possible winning strategy. Hence the interpretation of the universal quantifier is scope independent.

Above we assumed that each player has total information about previous moves of her opponent. Let us now change the rules of the game a bit by modifying our assumptions about scopal information. This new feature of the game will allow for a player to hide his move to others, so that a player might lack information about the previous moves of his opponent. For example, if Nature chooses an individual for the universal quantifier in (1) and I have no information about that choice, I can interpret the existential quantifier successfully (that is, I can have a winning strategy) only if the model would make the $\exists\forall$R true. That is, by lack of information, there is no winning strategy for Me unless Myself can choose an indivudual rendering the formula true regardless of My opponent's choice. But this is equivalent to the wide scope interpretation of the existential quantifier. Which means that we achieved a scope independent interpretation by purely semantic means, without invoking QR.

In section 1.2 we already pointed out that a wide scope interpretation may also come about by interpreting the indefinite as "specific", meaning that the speaker has a particular individual in mind. This specific reading, however, is still different from the one we have just described as scope independent. We already encountered a "mixed reading" under the label pseudo-scope in (49), a variant of which is (61):

(61) Each student has to hunt down every paper which shows that some condition proposed by Chomsky is wrong

Recall that we assume that *some condition* depends on *each student*, but not on *every paper*. To get the mixed reading we only have to assume that I have information about Nature's choice of a student but not about Nature's choice of a paper. This way, wide scope readings can be implemented without QR, and in particular without long distance QR across clause boundaries.

Summing up, we have seen that the $\exists\forall$R of (1) can be arrived at without QR, and so can the wide scope readings for indefinites that seem be non-restricted in scope. On the other hand, the inversed reading of (9) is still problematic: taking the structure of the sentence as determining the order of rule application, it would be necessary for Myself to make the first move of the game, hence the scope dependent reading cannot be derived. Hintikka therefore allows for a certain freedom of order in the interpretation of the sentence: within a certain domain the application seems to be free in that the order of quantifier evaluation is not necessarily restricted by syntactic structure.

Thus it might be allowed for Nature to begin, which seems to be almost equivalent to wide scope by QR. But not necessarily. Suppose we assume that identical variables can be replaced by a name, say ‘b’ as in the rules specified, *only* in the syntactic domain of a quantifier. For example, the evaluation of *his mother loves every boy* starts with the interpretation of the quantifier *every* and its restriction, yielding *his mother loves b, b a boy*. Since *his* is not in the syntactic scope of the QE, we would not be allowed to derive *b's mother loves b*, with the pronoun being interpreted by Nature. This way, *his* remains unbound, and what is called the cross-over condition in the QR theory is automatically accounted for by the restriction on variable interpretation formulated above.

In this section we derived sope-indepence as a lack of information. A formal way to implement this idea in the framework of ordinary Predicate Logic is by the use of so-called **Skolem-functions**, combinded with so-called **choice functions** that account for the restrictions of QEs. Skolem functions provide for a way to eliminate existential quantifiers from logical representations. This elimination, also called Skolemization, is performed by replacing every existentially quantified expression with a term $f(x_1, \ldots, x_n)$. The variables $x_1, \ldots, x_n$ correspond to variables introduced by universal quantifiers that have scope over the term to be eliminated. If there are no such variables, f is a zero-place function, i.e. an individual. Thus, the Skolemization of (3) is (62):

(62) a. $\forall x(\text{man}(x) \rightarrow (\text{woman}(f(x)) \wedge \text{love}(x, f(x))))$
b. $(\text{woman}(f) \wedge \forall x(\text{man}(x) \rightarrow \text{love}(x, f)))$

In order to account for the restriction of the quantifier in a compositional way, the method of Skolem function is combined with that of a choice function. A choice function takes a set as its argument and delivers an element of such a set. Combining this with Skolem functions, we assume the general format $f(X, x_1, \ldots x_n)$ with X as the restriction of the function f. In other words, all values of f must be in X. (There is technical complications with choice functions with an empty domain; this problem is ignored here, but see Winter (1997)). Accordingly, (62) now reads:

(63) a. $\forall x(\text{man}(x) \rightarrow \text{love}(x, f(\text{woman}, x)))$
b. $\forall x(\text{man}(x) \rightarrow \text{love}(x, f(\text{woman})))$

It is obvious that the difference between a scope-independent and a scope-dependent reading only resides in the presence or absence of a variable in the domain of f. Hence lack of information is simply modelled by lack of a (dependent, i.e. bound) variable.

Turning to the readings of (61), these can now easily be represented as:

(64) a. Each student x has to hunt down every paper y which shows that wrong(f(condition proposed by Chomsky))
b. Each student x has to hunt down every paper y which shows that wrong(f(condition proposed by Chomsky, x))
c. Each student x has to hunt down every paper y which shows that wrong(f(condition proposed by Chomsky, x, y))

Clearly, (a) is the wide scope reading, (b) the mixed reading, and (c) the narrow scope reading. A further analogy to the metaphors of Game Theory is also obvious: the sentences above are true iff there is such a function f which satisfies the above formula, which in turn holds iff I have a winning strategy.

The literature on Skolem- and choice-functions is immense, as many linguists have used one or the other in their analyses of scope-independence; cf. e.g. Liu (1990), Winter (1997), Geurts (2000), Stechow (2000), Sternefeld (2001a), Sternefeld (2010) and Steedman (2012) for further discussion.

4.2 Pseudo-scope-dependence of scope-independent DPs

Proper names like *Paul* pick out exactly one referent only if we restrict reference to "the Paul" under discussion; likewise definite descriptions like *the table* only refer to exactly one table in a small situation, and universal quantifiers like *every student* will normally not quantify over all students there are on this planet. One typically restricts the domain of interpretation to some subdomain of relevant individuals, cf. Fintel (1996).

What counts as relevant is by no means always obvious. E.g., in sentences like (65),

(65) Every man loves his wife

it is not clear whether or not the sentence should imply that every man has a wife. If not, we implicitly restrict the domain of *every* to the subset of married men.

Domain restrictions may also interact with scope inducing quantifiers in an interesting way. Consider (66):

(66) a. In most classes of Durham college, every student loves his teacher
b. Only one class was so bad that no student passed the exam

It seems that *every/no student* in (66) must be interpreted in such a way that the domain of quantification co-varies with the choice of a class, thus inducing some scope dependence we did not observe previously and which seems to contradict the general scope independence of *every* or *no* observed above (cf. also Stanley and Szabó (2000)).

Perhaps this is a side effect that should not bother us too much; the relativization to certain situations (the particular classes each defining such situations) is a kind of pseudo-scope dependency we may also observe in other expressions: To illustrate this effect, consider DPs like *the woman* which require some small domain with exactly one woman for successful reference. Whenever this condition is satisfied, the reference of the expression is completely fixed by the model, so that the semantic rules for the determiners *the* do not involve "a player making a choice". It follows that definite descriptions are not QEs. However, definite descriptions may well have scope. Thus, many linguists would say that in (67) the definite description is interpreted as scope-dependent on *every*:

(67) Every man adores the woman who loves him

The choice of a woman is dependent on that of the man. But this is only because the restriction of the determiner contains the pronoun *him* whose interpretation in turn depend on *every man*. This way, *any* expression containing a pronoun bound from outside by a scope-inducing quantifier can receive a "scope dependent" interpretation. Likewise definite descriptions may have scope with repect to modal operators. E.g.,

(68) The president might have lost the election

In the narrow scope reading for *the president*, (68) is necessarily false, whereas it is contingent in the wide scope reading. Again this is due to the interaction with the modal verb *might*, and the world dependence of the extension of the predicate *president*, but this fact alone does not, in my view, qualify definite descriptions as quantifiers, nor does it establish a scope dependency of *every*.

5 Other scope inducing expressions

Besides definite description, which we did not classify as quantifiers, there are many other expressions in natural language that are scope sensitive though not being GQs. Such expressions do have quantifying force and scope, hence must be considered quantifiers. E.g., this holds for all expressions that induce intensionality, as is the case for *might* in (68). Consider (69):

(69) John must buy a car

The interpretation of *must* involves the considerations of possible worlds in which John buys a car. More precisely, Nature tries to choose a possible world from a set of contextually relevant worlds that falsifies *John buys a car*; it is My turn then to evaluate *a car* in that world. If I am lucky, Nature cannot have a winning strategy, so that for any world w_N chosen by Nature the embedded sentence is true, i.e. Myself can select a car John buys in w_N. Thus, not only the interpretation of *buy* and *a car* depends on Nature's choice, it is also necessary for Me to find a car bought by John in w_N. Hence the QE *a car* is scope dependent on the interpretation of *must*.

Likewise, the modal verb *can* invites Me to choose a possible world that verifies its complement. Consider (70):

(70) Everyone can win

My choice of a possible world is dependent on Nature's choice for *everyone*. An implausible reading would imply a state of affairs in which everyone is a winner. This reading requires scope inversion: the domain of quantification of *everyone* is now influenced by the choice of a possible world and induces a kind of scope dependence of the kind already illustrated in (66). Clearly, *everyone* can be decomposed into the quantifier *every* and its domain of persons expressed by *one*. This domain can differ alongside with the possible worlds and the persons that exist in a world. This domain is world- and therefore scope-dependent, but not the quantifier *every* in and by itself.

According to possible world semantics, the interpretation of all intensional verbs involves the choice of one world or another. Viewed from this perspective, they are all quantificational. For example, the interpretation of *try* in a sentence like

(71) John tried to catch a fish

would involve an instruction for Nature to select a state of affairs s John intends to be in, such that, if Nature is going to win, John did not catch a fish in s. If successful the sentence is false, if Nature fails, it is true. The choice of the fish will of course be

Mine, but if there is no such fish, I have lost. By the same token, the scope independent interpretation would be one where I must find a fish which regardless of Nature's choice satisfies the condition that John wanted to catch it. This is equivalent to the wide scope interpretation of *a fish*.

We observe the same kind of ambiguity with other intensional verbs, e.g. those denoting propositional attitudes. As already observed by Lakoff (1970, p. 408), the QE may have wide scope in (72):

(72) Abdul believes that many men/few women like Baba Ghanoush

This reading seems to require QR out of the scope of *believe*. In the Game Theoretical construal, we do not need QR for this purpose. Of course, *believe* is a quantifier as it requires Nature to choose one of Abdul's belief worlds. If My choice of a set of men/women is independent of that world, I can only choose among the actual men/women. Hence an informationally impoverished *in situ* interpretation has the same truth conditions as a wide scope interpretation.

Besides intensional verbs, many other expressions of natural language can be classified as quantificational. Consider (73):

(73) When a man loves a woman ...

Here, *when* can have an interpretation in the sense of *whenever*; this has a clear quantificational interpretation, whether quantifying over events, over points of time, or over time intervals is immaterial in that respect, see also Lewis (1975). Further cases in point are adverbs like *always*, *seldomly*, *everywhere* and others; cf. "Nominal vs. Adverbial Quantification" in this handbook.

6 Distributivity and cumulation

We already encountered numeric quantifiers as potential counter-examples to GQT in sentence (19), repeated here in a sightly modified form as (74).

(74) *Two men met*: There is a set X such that two(X) and *[man](X) and met(X)

The new feature of (74) is the operator "*", a counterpart to the plural morphology of *man*. Semantically, this operator takes the extensions of predicates as input, e.g. the set Y of all men, and then forms the set of all non-empty subsets of Y. Moreover, we assume that in formulas like (74), *[] is the characteristic function of that set. A sentence like *Two men laughed* could then be formalized as:

(75) There is a set X such that two(X) and *[man](X) and *[laugh](X)

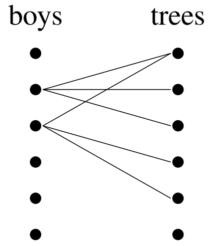
The plural operation *[...] is a type shifting operation which is inherently quantificational, as the following equivalence holds:

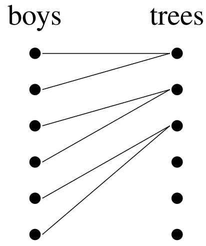
(76) *[α](X) is true iff **for all** x in X, $\alpha(x)$ is true.

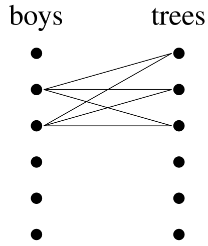
In this section, we add another aspect to the analysis, namely that of scope. It is often observed that sentences like

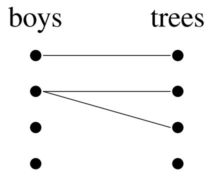
(77) Two boys climbed three trees

can be (at least) four way ambiguous (in English), as illustrated by the typical models in (78):

(78) a.


b.


c.


d.


In order to simplify the discussion, it is assumed that the numerals *two* and *three* in (77) are interpreted as *exactly two* and *exactly three* (cf. “Quantity Implicature” in this handbook).

The four readings can be associated with the following semi-formal representations (cf. Langendoen (1978) and Scha (1984)).

(79) a. $\exists X(\text{two}(X) \wedge {}^*[\text{boy}](X) \wedge \forall x \in X :$
$\exists Y(\text{three}(Y) \wedge {}^*[\text{tree}](Y) \wedge \forall y \in Y : \text{climb}(x,y)))$

b. $\exists Y(\text{three}(Y) \wedge {}^*[\text{tree}](Y) \wedge \forall y \in Y :$
$\exists X(\text{two}(X) \wedge {}^*[\text{boy}](X) \wedge \forall x \in X : \text{climb}(x,y)))$

c. $\exists X(\text{two}(X) \wedge {}^*[\text{boy}](X) \wedge \exists Y(\text{three}(Y) \wedge {}^*[\text{tree}](Y) \wedge$
$\forall x \in X : \forall y \in Y : \text{climb}(x,y)))$

d. $\exists X(\text{two}(X) \wedge {}^*[\text{boy}](X) \wedge \exists Y(\text{three}(Y) \wedge {}^*[\text{tree}](Y) \wedge$
$\forall x \in X : \exists y \in Y(\text{climb}(x,y)) \wedge \forall y \in Y : \exists x \in X(\text{climb}(x,y))$

(79a) represents the linear reading in which the subjects has scope over the object. (79b) is the inverse reading (which seems to be unavailable for most speakers of German). (79c) is called the strong reciprocal reading, and (79d) is called the cumulative reading, see the article “Plurality and Cumulation: Dutch Computers” in this handbook.

In this context it is important to observe that the reciprocal reading and the cumulative reading are scopeless. Neither quantifier has scope over the other: this applies for the sequence of existential quantiers in (c) and (d) as well as to the sequence of universal quantifiers in (c); concerning (d), the quantifiers that originate from different QE are intertwined.

As the reader may verify, standard GQT can only account for the linear reading (a) and the scope reversal in (b). The compositional derivation of (c) and (d) is a problem of its own; I once proposed that this can be achieved by two types of shifing operators that turn relations over indiviuals into relations over sets: one of them, denoted by ++ is reciprocal, the other one, denoted by ** is cumulative, cf. Sternefeld (1998) for details. Given appropriate definitions of ++ and **, the resulting presentations in (80) are logically equivalent to (79c) and (79d), respectively:

(80) a. $\exists X(\text{two}(X) \wedge {}^*[\text{boy}](X) \wedge \exists Y(\text{three}(Y) \wedge {}^*[\text{tree}](Y) \wedge^{++} [\text{climb}](x, y)))$
b. $\exists X(\text{two}(X) \wedge {}^*[\text{boy}](X) \wedge \exists Y(\text{three}(Y) \wedge {}^*[\text{tree}](Y) \wedge {}^{**}[\text{climb}](x, y)))$

This solution is in the spirit of Link (1983), as the source of the ambiguity can ultimately be derived by operations on the verb or the VP. However, Link does not assume second order quantification but instead enriches his ontology with plural objects, called sums, which are individuals rather than sets. This way, we can stay consistent with the standard theory of GQT, except that we have to slightly modify the semantics of the quantifiers (cf. Link (1987)). For example, the *three* in such a modified GQT does no longer pick out all sets with three elements as its restriction, but only sets with three entities that are not sums. In traditional terminology, we count "indidividuals" but not sums. Such primitive individuals are called atoms, and the distinction between atoms and sums is part of Link's ontology.

An alternative approach locates the ambiguity in the analysis of the QEs themselves. E.g., Scha (1981) distinguishes between distibutive, collective, and cumulative readings of quantifiers. The question then arises how this purported lexical ambiguity can contribute to the sentence meaning in a compositional way. This problem is addressed within GQT by Van der Does (1998), who lifts GQ into second order quantifiers which express the required distinctions by corresponding different analyses of the scope of the quantifier (cf. op. cit. p. 248f for an analysis of numerals). Accordingly, the scope of a quantifier is no longer a set, but a set of sets the structure of which is either distributive, collective, or cumulative, corresponding to the different operations on the VP in Link's analysis. In consequence, GQs do not denote sets of sets, but sets of sets of sets.

This shift of types makes GQT even more complex, in yet unexplored ways, and seems to contrast sharply with the simplicity of the analyses in (80). Taken to the extreme, the different views can be characterized by saying that in one theory we simply *stipulate* that there are only very view (perhaps ultimately only three) "real" quantifiers, whereas in GQT we start off with a hugh universe of potential quantifiers and would need very many axioms that exclude the ones that apparently don't exist. E.g., as observed above, such axioms must exclude numerals that count sums instead of atoms, or we would have to exclude universal quantification over sets, which seems to be non-existent in natural language.

Let us now return to the scope enducing force of the hidden star-operation illustrated in (76). The following example from Reinhart (1997) has provoked much discussion (see the article "Wide Scope Indefinites: Dead Relatives" in this handbook):

(81) If three relatives of mine die, I will inherit a house

This sentence is ambiguous: (a) if the number of my dead relatives reaches the number three, then I inherit a house (regardless of which relatives pass away); and (b) there are three particular relatives such that if they all die I will inherit a house.

The (b)-reading seems to be particularly cumbersome, as it seems to require wide scope of the QE across the clause boundary of the conditional. But even if QR across that boundary were permitted, the truth conditions come out wrong, as the reader might easily verify, by comparing the incorrect representation in (82a) with the correct one in

(82b):

(82) a. There is a set X such that three(X) and *[relative-of-mine](X) and *[$\{x$: if die(x) then I inherit a house$\}$](X)

b. There is a set X such that three(X) and *[relative-of-mine](X) and if *[die](X) then I inherit a house

Suppose the wide scope of the indefinite is interpreted by *in situ* scope independence. The conditional urges Nature to choose a possible world verifying the antecedent of the conditional, but the interpretation of the QE does not depend on that choice, hence the QE is interpreted outside the conditional: Myself must select a set of individuals among My relatives in the actual world. But *die* now applies to the set X inside the *if*-clause so that the incorrect truth conditions corresponding to (82a) can no more be derived. For more on the scope independence of restrictions, cf. also Sternefeld (2010) and the article "Scope and Indices: Bäuerle's Paradox and Related Issues" in this handbook.

7 Dynamic systems

The following sentences pose a challenge to most theories of quantification and scope:

(83) a. A man entered the room. He whistled.

b. If a man is in Athens, he is not in Rhodes

c. When a man loves a woman, I know exactly how he feels

Trying to interpret the pronoun *he* as anaphoric to *a man* turns out problematic because, as things stand, the scope of quantifiers is completely determined by the "bracketing", i.e. the syntax, of these constructions. This is part and parcel of any semantic theory of natural language, as any such theory specifies a procedure that translates syntactic into semantic structure. Given the syntactic structure and the theory of Transparent Logical Form, the scopal domain of a QE is exactly its syntactic c-command domain (at LF). This domain is usually restricted by the minimal clause that contains the quantifier in question. Hence the pronoun cannot be in the scopal domain of *a man*, hence cannot be bound, contrary to intuition. Moreover, the scope cannot simply be extended to the following clause or sentence: If we were to raise the quantifier *a man* across the conditionals in (83b,c) by QR, we get the wrong truth conditions: since a conditional is true as soon as the antecedent is false, (83b,c) would be true as soon as we can find a man who is not in Athens or a man who does not love a woman.

The problem is extensively discussed in the contributions on **donkey anaphora** in this handbook (in particular: "Donkey anaphora: Farmers and Bishops") and the theories discussed there are highly relevant to the concepts of quantification and scope.

Two types of theories have been proposed. The first takes up basic ideas from Lewis (1975) in saying that indefinite DPs are not quantifiers but rather correspond to open propositions (so that *a man* = man(x)) with a free variable that becomes bound by operators determined by DPs environment, for example by adverbs of quantification. One such proposal has been worked out by Heim (1982) in her dissertation, another slightly different one by Hans Kamp (1981). The latter has later become known as **Discourse Representation Theory** (DRT, cf. also Kamp and Reyle (1993)), the former as **file**

change semantics, see Heim (2002). Since Lewis, the basic common idea is that the conditional *if… then* or the temporal conditional *when* are **unselective binders** which have universal force over such variables, so that (83b) effectively quantifies over all men in Athens and (83c) quantifies over all pairs of men and women. Technically, it is not the QE having scope over the pronoun; it's rather the conditional expressions *if… then* and *when* that semantically binds the variable x and a coindexed pronoun. This idea can be generalized from adverbs and conditionals to all kinds of scope inducing expressions, including universal quantifiers.

The second type of theory was first formulated by Richard Smaby (1979) and has become popular in so-called **Dynamic Predicate Logic** (cf. Groenendijk and Stokhof (1991), see also Gamut (1991) Chapter 7, for a nice introduction and a comparison between Dynamic Predicate Logic and DRT). Here indefinites still are quantifiers, the interpretation of a sequence of sentences is interpreted as conjunction of two clauses, and it is still true that the quantifiers do not c-command the pronouns and are interpreted in the usual way. However, the interpreation of indefinites involve something in addition which is intended to interpret free variables (i.e. the translation of pronouns) in a non-standard way. Technically, this is an additional assignment function. Speaking figuratively in terms of Game Theory, this additional mechanism causes a third party to come into play: this third player can memorize My choice of an individual and will pass that information on to the interpretation of subsequent clauses. Hence the interpretation of the second clauses in (83) is dynamically dependent on the output of the interpretation of the first clauses, which makes conjunction and other connectives a "dynamic" operation. When it comes to the interpretation of the pronoun, it's the third player who does the job of providing a value for *he* in the next sentence. Due to the dynamic interpretation of conjunction, the rules of the game have to be modified so that Nature (the Falsifier) cannot simple pick out a conjunct, as the second conjunct is interpreted as a result of the interpretation of the first. Thus the Verifier (= Myself) first has to evaluate the first conjunct, and if successful, passes the second conjunct to the Falsifier and the third player. This makes conjuction asymmetric and "dynamic".

We thus need some special rules governing the behavior of the players. For example, a sequence of sentences like

(84) No man entered the room. He whistled.

does not seem to allow the anaphoric construal of *he*; this can be made follow from the assumption that the negation contained in the first sentence,

(85) It's not the case that a man entered the room. He whistled.

will effectively make the third player forget everything he registered about the choice (Nature's choice) inside the scope of negation. (But see the remarks on "Telescoping" at the end of this section.)

As demonstrated by Smaby (1979), asymmetric conjunction also solves the problem posed by conditionals, namely that the existential quantifier seems to gain universal force when binding a pronoun in the consequent of the conditional:

(86) If a man entered the room, he whistled
= Any/every man who entered the room, whistled

How come? As is well known from propositional logic, a conditional like (A implies B) is equivalent to “it is not the case that (A and not B)”. Now suppose this is indeed the semantics of the conditional, with the important switch that the *and* is interpreted in a dynamic way, as described above. That is, we have to negate (87):

(87) A man entered the room. He didn’t whistle.

The anaphoric relation is perfectly possible here, because there is no negation in the antecedent that would block the memory of the third party. The negation of (87), namely (88),

(88) It is not the case that there is a man who entered the room and who did not whistle.

is equivalent to the universal proposition that every man who entered the room, whistled, which in turn is equivalent to (86). We thus solved the problem of universal force without having to change anything dramatic in the semantics of the quantifiers or the conditional. (For more on the problem of binding without c-command, see “Donkey anaphora: Farmers and Bishops” and E-type pronouns” in this handbook.)

What we can learn from dynamic systems is this: there is a way to interpret binding independently of c-command, without QR, *in situ*, in violation of the Scope Principle, and in a compositional way. Moreover, Dynamic Predicate Logic is not the only way of doing so, cf. e.g. Sternefeld (2001b), Barker and Shan (2014) and Klein and Sternefeld (2013). This is an important insight, as it has repercussions in many other areas of linguistic research, in particular when it comes to encode scope or scopelessness without having to depart from surface syntax and without alluding to different levels of representation. See e.g.Bott and Sternefeld (2017) where dynamic binding is crucial to obtain surface compositionality, or AnderBois *et al.* (2015) in their account of presupposition and appositives.

Returning to (84), we proposed that negative quantifiers cannot extend scope, as the third player cannot recall My choice beyond the sentence boundary, and the same should hold for Nature’s choice when interpreting universal quantifiers. However, there might be circumstances in which this restriction can be relaxed. For example, it seems as if the universal quantifier can gain scope out of its minimal clause (a process also called “**telescoping**”) in (89):

(89) a. Every Italian$_i$ loves his mother. He$_i$ adores her.
b. Each candidate$_i$ for the space mission meets all our requirements. He$_i$ has a Ph.D. in astrophysics and extensive prior flight experience (from Roberts (1987, p. 718))
c. The picture of his$_i$ mother that every soldier$_i$ kept wrapped in a sock was not much use to him$_i$

It’s an open question whether or not these data should be considered cases of “real variable binding”, but if they are, they can easily be accounted for in a dynamic system, cf. Poesio and Zucchi (1992) or Sternefeld (2017).

In consequence, pronominal reference cannot be used as a water-proof test for certain “readings”. For example, it has often been claimed (cf. Ruys and Winter (2011)) that a successful understanding of (90) guarantees that the speaker/hearer has a ∃∀R in mind.

(90) Every boy climbed a tree$_i$. It$_i$ was tall.

But experimental evidence teaches that speakers may well be able to interpret *a tree* as scope dependent on *every*, but nonetheless extend the scope of *a tree* beyond its clause (understanding that each tree climbed by a boy was tall). Cf. also:

(91) a. Every man loved a woman. She was tall.
b. Every boy climbed a tree. The tree was tall.

As observed by Bott and Radó (2007), (91b) is still ambiguous, the surprising reading being the one with *every boy* having scope over *a tree* and over the domain restriction for the determiner *the*, as was discussed with reference to (66) of section 4.2. This way, the determiner's domain adjusts to small situations with only a single boy and a single tree. The determiner picks out "the (one) tree" of each climbing situation, but the total number of trees may well be greater than one.

Summarizing so far, it is far from clear how scope is related to syntactic structure and how the problematic apparent wide scope readings come about.

8 Psycholinguistic research

There is a growing psycholinguistic literature on the quantification and scope; the main question is how quantifiers, quantifier scope and pronominal binding is processed online. Less prominent topics are how quantifiers are acquired in children (cf. Geurts (2003), Musolino and Lidz (2006) Hunter and Lidz (2013)) and how quantifiers relate to the neuro-anatomic underpinnings of number sense in cognitive neuro-science (cf. Clark and Grossman (2007)). I cannot do justice here to this line of research but would like to mention only a few further readings. Most relevant in the present context is Tunstall (1998) and Filik *et al.* (2004) who investigate the processing of the ∃∀R/∀∃R ambiguity in doubly quantified sentences, the former focussing on the difference in distributivity between *each* and *every* whereas the latter focussing on different grammatical functions (direct object, indirect object) of the QEs. Bott and Radó (2009) investigate the difference between *each* and *all*, as illustrated in (92):

(92) a. Some airline serves all continents
b. Some airline serves each continent

Ioup (1975) already observed that *each* permits a wide scope reading much more easily than *all*. This is tested experimentally for various constructions of German.

Another aspect of QEs is their **complexity**: Some QE seem to take longer than others in comprehension experiments. As discussed above, a quantifier may be upward entailing (i.e. license inferences from subsets to supersets) or downward entailing (i.e. license inferences from supersets to subsets; see above in the context of Negative Polarity). A paper that investigates the interaction between such quantifiers is Geurts and van der Slik (2005), where one of the results is that sentences containing both upward and downward entailing quantifiers are more difficult than sentences with upward entailing quantifiers only. The authors take this as evidence for the theses that downward quantifiers are generally more difficult than upwars ones. They take this as evidence in favor of GQT, as the properties in question are easily definable in that framework. However, as already mentioned above, I do not see that the relevance of those properties might rescue GQT as a theory of quantification, as the upward/downward entailing property is not specific to quantifiers and, in general, as the authors themselves show, inferences from subsets to supersets are generally easier than inferences in the opposite direction (cf. Szymanik (2009)). Moreover, it has been shown recently by Schlotterbeck (2016) that in some contexts under some conditions downward entailing quantifiers are in fact quicker than upward ones. The author's explanation amounts to saying that the downward ones have to be decomposed, with negation as one of its components. Now, sentences with overt or covert negation are generally more difficult to understand than positive ones, but this effect can pragmatically be overrun in certain contexts. If so, we arrive at the same result as with negative polarity, namely that GQT cannot contribute to a genuine explanation of the phenomena under discussion.

Another interesting issue concerns the verification procedure for QEs. By a verification procedure we mean an algorith that can decide the truth or falsity of a sentence. Mathematically, there are many different ways to characterize one and the same quantifier. But which one of them, if any, has some kind of "cognitive reality"? Pietroski *et al.* (2009), Lidz *et al.* (2011), and Schlotterbeck (2016) attack this question by investigating different verification strategies for sentences with *most*. See also the discussion of *most* and its decomposition in Hackl (2009).

At the end of this article, I would like to thank Oliver Bott, Fabian Schlotterbeck, and Lisa Matthewson for helpful comments. Needless to say that in each section above many questions remained unresolved, and I could not do justice to the immense amount of work that has been done in the field. For the discussion of even more data and analyses I refer the reader to yet another handbook article, Szabolcsi (2012), and to a booklength survey by the same author, see Szabolcsi (2010).

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